RELATION BETWEEN CONFINEMENT AND HIGHER SYMMETRY RESTRICTIONS FOR COLOR PARTICLE MOTION

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Quantum operators of coordinates and momentum components of a particle in Minkowski spacetime belong to the generalized Snyder-Yang algebra in the general case and produce a quantum phase space. Assuming the O(2,6) invariance in the quantum phase space of a color particle the equation of motion is obtained, which contains a oscillator rising potential. The existence of the oscillator potential can explain the confinement of quarks. A parameter of the potential and quantum constants with the dimensions of mass and length are estimated.

Keywords: symmetry; quantum constant; quantum phase space; quark; confinement

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Now it is generally accepted that the QCD is the theory of strong interaction of quarks and gluons. As is known the QCD operates with the quantum color fields of quarks and gluons defined in the conventional four dimensional Minkowski spacetime $M_{1,3}$ [1, 2]. The QCD has considerable verification at high interaction energies, however some problems remain unsolved in the low energy region such as confinement of color particles and breaking of chiral invariance of the massless QCD lagrangian.

In the present letter we consider the possibility for resolving the confinement problem with the help of higher symmetry restrictions for motion of a color particle. To do this would require an additional prerequisite, namely, the O(2,6) invariance in a phase space of a color particle [3].

Since coordinates and momenta of a quantum particle can be noncommutative in the general case, let us start with the generalized Snyder-Yang algebra (GSYA) to be considered in the following form [4, 5, 6, 7]:

$$
\begin{aligned}
&[F_{ij}, F_{kl}] = i(g_{jk}F_{il} - g_{ik}F_{jl} + g_{il}F_{jk} - g_{jl}F_{ik}),\\
&[F_{ij}, p_k] = i(g_{jk}p_i - g_{ik}p_j),\\
&[F_{ij}, q_k] = i(g_{jk}q_i - g_{ik}q_j),\\
&[F_{ij}, I] = 0, \quad [p_i, q_j] = i(g_{ij}I + \kappa F_{ij}), \qquad (1)\\
&[p_i, I] = i(\mu^2 q_i - \kappa p_i), [q_i, I] = i(\kappa q_i - \lambda^2 p_i),\\
&[p_i, p_j] = i\mu^2 F_{ij}, \quad [q_i, q_j] = i\lambda^2 F_{ij},
\end{aligned}
$$

where $c = \hbar = 1$, $i, j, k, l = 0, 1, 2, 3$. The new quantum constants μ and λ have dimensionality of mass and lenght correspondingly. The constant κ is dimensionless in the natural system of units.

By applying the algebra (1) to description of color particles the condition $\kappa = 0$ should be imposed. Actually it is known the nonzero κ leads to the $CP-$violation [6], but the strong interactions are invariant with respect to the $P-$, $C-$ and $T-$transformations on the high level of precision. Moreover for color particles we use the relation $\mu\lambda = 1$[1]. Thus we obtain the reduction of the GSYA to the special Snyder-Yang algebra (SSYA) with $\mu\lambda = 1$ and $\kappa = 0$ for strong interaction color particles. Denoting μ as μ_c and λ as λ_c we show the following commutation relations besides of the usual commutation relations with Lorentz group generators for the GSYA, which are written above (see Eqs.(1)).

$$
\begin{aligned}
&[p_i, q_j] = ig_{ij}I, [p_i, I] = i\mu_c^2 q_i, [q_i, I] = -i\lambda_c^2 p_i,\\
&[q_i, q_j] = i\lambda_c^2 F_{ij}, \quad [p_i, p_j] = i\mu_c^2 F_{ij}. \qquad (2)
\end{aligned}
$$

Taking into account difficulties arised to prove the confinement on the basis of the QCD first principles we assume that this phemomenon is connected with high symmetry of the nonperturbative OCD interaction. Or the nonperturbative OCD interaction have the property of the approximated high symmetry. In

[1] In Ref.[7] this relation had been replaced with the relation $\lambda = 0$.

any case although a detailed interconnection cannot be revealed (or this is the very difficult task) we turn from Poincare symmetry in Minkowski spacetime to an inhomogeneous O(2,6) symmetry in a phase space of a color particle [3].

In this way we consider a generalized theory for color particles, when coordinates and momenta are on equal terms and form an eight dimensional phase space: $h = \{h^A | h^A = q^\mu, A = 1,2,3,4,\ \mu = 0,1,2,3,\ h^A = \lambda^2 p^\mu, A = 5,6,7,8,\ \mu = 0,1,2,3\}$. So a generalized lenght square

$$L^2 = h^A h_A, \tag{3}$$

and a generalized mass square

$$M^2 = P^A P_A, \tag{4}$$

are invariant under the O(2,6) transformations, where $h_A = g_{AB} h^B$, $g_{AB} = g^{AB} = diag\{1,-1,-1,-1,1,-1,-1,-1\}$, $P = \{P^A | P^A = p^\mu, A = 1,2,3,4,\ \mu = 0,1,2,3,\ P^A = \mu^2 q^\mu, A = 5,6,7,8,\ \mu = 0,1,2,3\}$. Values of the constants μ and λ are proportional to the μ_c and λ_c values, correspondingly. As a preliminary we use below $\mu = \mu_c$ and $\lambda = \lambda_c$.

Thus we propose that for strong interacting color particles the generalized mass squared has the physical meaning:

$$\begin{aligned} dM^2 &= (dp_0)^2 - (dp_1)^2 - (dp_2)^2 - (dp_3)^2 \\ &+ \mu_c^4 (dq_0)^2 - \mu_c^4 (dq_1)^2 - \mu_c^4 (dq_2)^2 - \mu_c^4 (dq_3)^2 \\ &= (dm)^2 + \mu_c^4 (ds)^2. \end{aligned} \tag{5}$$

An important point is that the coordinates q^μ and the momenta p^μ are the quantum operators satisfied Eqs.(2) in the frame of this approach.

Under these conditions a new Dirac type equation for a spinorial field ψ have the following form:

$$\gamma^A P_A \psi = M \psi, \tag{6}$$

where γ^A are Clifford numbers for a spinorial O(2,6) representation, i.e.

$$\gamma^A \gamma^B + \gamma^B \gamma^A = 2 g^{AB}. \tag{7}$$

One can take the product of Eq.(6) with $\gamma^A P_A + M$, then the following equation for ψ is obtained

$$\begin{aligned} &(p^i p_i + \mu^4 q^i q_i + i\mu^2 C^0 I + \\ &+ i\mu^2 \Sigma_{i<j} C^{ij} F_{ij}) \psi = M^2 \psi, \end{aligned} \tag{8}$$

where

$$\begin{aligned} C^0 &= \gamma^1 \gamma^5 g_{00} + \gamma^2 \gamma^6 g_{11} \\ &\quad + \gamma^3 \gamma^7 g_{22} + \gamma^4 \gamma^8 g_{33}, \\ C^{01} &= \gamma^1 \gamma^2 + \gamma^5 \gamma^6, \\ C^{02} &= \gamma^1 \gamma^3 + \gamma^5 \gamma^7, \\ C^{03} &= \gamma^1 \gamma^4 + \gamma^5 \gamma^8, \\ C^{12} &= \gamma^2 \gamma^3 + \gamma^6 \gamma^7, \\ C^{13} &= \gamma^2 \gamma^4 + \gamma^6 \gamma^8, \\ C^{23} &= \gamma^3 \gamma^4 + \gamma^7 \gamma^8. \end{aligned} \tag{9}$$

Eq.(8) contains the oscillator potential, which restricts the movement of a color quark. Besides that we broke the inhomogeneous O(2,6) symmetry to the inhomogeneous O(1,3) symmetry.

Let us consider other consequences of this approach for specific color quark properties. From the relations (2) it immediately follows nonzero uncertainties for results of simultaneous measurements of quark momentum components. For instance, let $\psi_{1/2}$ is a quark state with a definite value of its spin component along the third axis. Consequently, $[p_1, p_2] = i\mu_c^2/2$, thus

$$\Delta p_1 \Delta p_2 \geq \mu_c^2/4 \tag{10}$$

and if $\Delta p_1 \sim \Delta p_2$, one gets $\Delta p_1 > \mu_c/2$, $\Delta p_2 > \mu_c/2$. We see the generalized quark momentum components $p_{\perp 1,2}$ cannot be measured better than tentatively one-half a value of μ_c.

One can get an independent estimation of the μ_c value using the quark equation (8). Let us assume that two parameters with dimensions of mass entered into the Eq.(8) (M^2 and p^2) are equal to the current and constituent masses squared, respectively. So Eq.(8) indicates that the canonical relation $p^2 = M^2$ for a current quark should be transform to $p^2 = M^2 + \Delta^2$ for a constituent quark, where $m^2 = M^2 + \Delta^2$ is a constituent mass squared. To estimate a μ_c value with the help of a constituent quark mass m and a current quark mass M values a ground state ψ_0 in a meson is considered neglecting the orbital angular momentum contribution $L\psi_0$. By this means in Ref.[7] it has been obtained that the μ_s value is of the order of 0.2 GeV. However in so doing another equation for a generalized quark

has been used, as compared with Eq.(6). As a consequence uncertainty takes place, when we estimate an additional contribution to a mass of a constituent quark.

It is known quark confinement is investigated in the frame of different approaches such as the lattice QCD, Schwinger-Dyson equations, massive transverse gluons, potential models [8, 9, 10, 11]. Let us estimate the μ_c value with the help of a value of a confinement rising potential coefficient. This procedure gives a result within certain limits. One can see from Eq.(8) that the coefficient of the oscillator confinement potential for a color particle is equal to μ_c^4. In the one particle potential approach this coefficient is connected with the so-called "string tension" σ typically as $\mu_c^4 = \sigma^2/4$, where σ varies from 0.19 GeV^2 to 0.21 GeV^2 [12]. Hence $0.31GeV < \mu_c < 0.32GeV$ and $0.62Fm < \lambda_c < 0.64Fm$. It is assumed that the conceptions of an asymptotical oscillator potential and a constituent quark are applicable in a confinement region.

Here we consider a few consequences of the generalized O(2,6) symmetry in the quantum phase space of a color particle, in particular, for the color particle confinement. The further investigation of properties of solutions of the equations (6) and (8) is the important objective of this approach and under consideration now.

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Relation between confinement and higher symmetry restrictions for color particle motion

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Quantum operators of coordinates and momentum components of a particle in the Minkowski spacetime can belong to the generalized Snyder-Yang algebra and produce a quantum phase space with three new constants in the general case. With account for the O(2,6) invariance in the quantum phase space of a color particle the equation of motion is obtained, which contains an oscillator rising potential. The presence of the oscillator potential can simulate a confinement of a color particle. A parameter of the oscillator potential is estimated and a relationship between current and constituent quark masses is obtained.

Keywords: spacetime symmetry; quantum constant; quantum phase space; quark; confinement

PACS: 11.30.Ly; 11.90.+t; 12.90.+b

1. Introduction

Now it is generally accepted that QCD is the theory of strong interaction of quarks and gluons. As is known QCD operates with quantum color fields of quarks and gluons defined in the conventional four dimensional Minkowski spacetime $M_{1,3}$ [1, 2]. QCD has considerable verification at high interaction energies, however some problems remain unsolved in the low energy region, for instance, a confinement of color particles and a violation of chiral invariance of the massless QCD lagrangian.

It is well known that now the quark confinement is investigated in the frame of different approaches such as lattice QCD, Schwinger-Dyson equations, massive transverse gluons, potential models, *etc* [3, 4, 5, 6]. In the present letter we consider a model for resolving the confinement problem with the help of higher symmetry restrictions for a motion of a color particle. To do this would require an additional prerequisite, namely, the O(2,6) invariance in a phase space of a color particle, which have been proposed in ref.[7]. Besides we use the quantum phase space with three fundamental constants additional to the standard ones c and $\hbar$ [8, 9, 10].

2. Restrictions for color particle motion based on higher symmetry in a quantum phase space

Since coordinates and momentum components of a quantum particle can be noncommutative in the general case, let us start with the generalized Snyder-Yang algebra (GSYA) to be considered in the following form [8, 9, 10, 11]:

$$
\begin{aligned}
&[F_{ij}, F_{kl}] = i(g_{jk}F_{il} - g_{ik}F_{jl} + g_{il}F_{jk} - g_{jl}F_{ik}),\\
&[F_{ij}, p_k] = i(g_{jk}p_i - g_{ik}p_j),\\
&[F_{ij}, q_k] = i(g_{jk}q_i - g_{ik}q_j),\\
&[F_{ij}, I] = 0, \quad [p_i, q_j] = i(g_{ij}I + \kappa F_{ij}), \qquad (1)\\
&[p_i, I] = i(\mu^2 q_i - \kappa p_i), [q_i, I] = i(\kappa q_i - \lambda^2 p_i),\\
&[p_i, p_j] = i\mu^2 F_{ij}, \quad [q_i, q_j] = i\lambda^2 F_{ij},
\end{aligned}
$$

where $c = \hbar = 1$, F_{ij}, p_i, x_i are the generators of the Lorentz group and the operators of momentum components and coordinates, correspondingly, I is the "identity" operator, $i, j, k, l = 0, 1, 2, 3$. The new quantum constants μ and λ have dimensionality of mass and lenght correspondingly. The constant κ is dimensionless in the natural system of units.

By applying the algebra (1) to the description of color particles the condition $\kappa = 0$ can be imposed. Actually it is known the nonzero κ leads to the $CP-$violation [10], but strong interactions are invariant with respect to the $P-$, $C-$ and $T-$transformations on the high level of precision. Moreover for color particles one can use the relation $\mu\lambda = 1$[1]. In this case we obtain the reduction of GSYA to the special Snyder-Yang algebra (SSYA)

[1] In Ref.[11] the relation $\lambda = 0$ is used instead of $\mu\lambda = 1$.

with $\mu\lambda = 1$ and $\kappa = 0$ for strong interaction color particles. Denoting μ as μ_c and λ as λ_c we write the following commutation relations without the standard commutation relations with Lorentz group generators, which are shown for the GSYA above (see eqs.(1)).

$$[p_i, q_j] = ig_{ij}I, [p_i, I] = i\mu_c^2 q_i, [q_i, I] = -i\lambda_c^2 p_i,$$

$$[q_i, q_j] = i\lambda_c^2 F_{ij}, \quad [p_i, p_j] = i\mu_c^2 F_{ij}. \tag{2}$$

We take into account difficulties arised when one try to prove the confinement on the basis of the QCD first principles, so we simulate this phemomenon with the help of an assumed high symmetry of the nonperturbative OCD interaction. We suppose that the nonperturbative OCD interaction have the property of an approximated or exact higher spacetime symmetry beyond the Poincare symmetry. In our model we turn from the Poincare symmetry in the Minkowski spacetime to the inhomogeneous O(2,6) symmetry in a phase space of a color particle [7].

In this way we consider the generalized model for a color particle motion, when coordinates and momenta are on equal terms and form an eight dimensional phase space: $h = \{h^A | h^A = q^\mu, A = 1,2,3,4, \mu = 0,1,2,3, h^A = \tau p^\mu, A = 5,6,7,8, \mu = 0,1,2,3\}$. $P = \{P^A | P^A = p^\mu, A = 1,2,3,4, \mu = 0,1,2,3, P^A = \sigma q^\mu, A = 5,6,7,8, \mu = 0,1,2,3\}$. The constants τ and σ have dimensions of length and mass square, correspondingly. Their values can be chosen on the phenomenological ground or with the help of some functions of the quantum constants μ, κ and λ. So the generalized lenght square

$$L^2 = h^A h_A, \tag{3}$$

and the generalized mass square

$$M^2 = P^A P_A, \tag{4}$$

are invariant under the O(2,6) transformations, where $h_A = g_{AB}h^B$, $g_{AB} = g^{AB} = diag\{1,-1,-1,-1,1,-1,-1,-1\}$.

Thus we propose that for strong interacting color particles the generalized differential mass squared has the physical meaning:

$$dM^2 = (dp_0)^2 - (dp_1)^2 - (dp_2)^2 - (dp_3)^2$$

$$+\sigma^2(dq_0)^2 - \sigma^2(dq_1)^2 - \sigma^2(dq_2)^2 - \sigma^2(dq_3)^2$$

$$= (dm)^2 + \sigma^2(ds)^2. \tag{5}$$

An important point is that the coordinates q^μ and the momentum components p^μ are the quantum operators satisfied eqs.(1) or eqs.(2) in the frame of this approach.

Under these conditions the new Dirac type equation for a spinorial field ψ has the following form:

$$\gamma^A P_A \psi = M\psi, \tag{6}$$

where γ^A are the Clifford numbers for the spinorial O(2,6) representation, i.e.

$$\gamma^A\gamma^B + \gamma^B\gamma^A = 2g^{AB}. \tag{7}$$

One can take the product of eq.(6) with $\gamma^A P_A + M$ and apply eqs.(1), then the following equation for ψ can be obtained

$$(p^i p_i + \sigma^2 q^i q_i + 2\Sigma_{i<j} S^{ij} F_{ij} +$$

$$+ 2\sigma S^0 I)\psi = M^2\psi, S^0 = \frac{i}{2}C^0, S^{ij} = \frac{i}{2}C^{ij}, \tag{8}$$

where

$$C^0 = \gamma^1\gamma^5 g_{00} + \gamma^2\gamma^6 g_{11} + \gamma^3\gamma^7 g_{22} + \gamma^4\gamma^8 g_{33},$$

$$C^{01} = \gamma^1\gamma^2\mu^2 + \gamma^5\gamma^6\sigma^2\lambda^2 + (\gamma^1\gamma^6 - \gamma^2\gamma^5)\sigma\kappa,$$

$$C^{02} = \gamma^1\gamma^3\mu^2 + \gamma^5\gamma^7\sigma^2\lambda^2 + (\gamma^1\gamma^7 - \gamma^3\gamma^5)\sigma\kappa,$$

$$C^{03} = \gamma^1\gamma^4\mu^2 + \gamma^5\gamma^8\sigma^2\lambda^2 + (\gamma^1\gamma^8 - \gamma^4\gamma^5)\sigma\kappa,$$

$$C^{12} = \gamma^2\gamma^3\mu^2 + \gamma^6\gamma^7\sigma^2\lambda^2 + (\gamma^2\gamma^7 - \gamma^3\gamma^6)\sigma\kappa,$$

$$C^{13} = \gamma^2\gamma^4\mu^2 + \gamma^6\gamma^8\sigma^2\lambda^2 + (\gamma^2\gamma^8 - \gamma^4\gamma^6)\sigma\kappa,$$

$$C^{23} = \gamma^3\gamma^4\mu^2 + \gamma^7\gamma^8\sigma^2\lambda^2 + (\gamma^3\gamma^8 - \gamma^4\gamma^7)\sigma\kappa. \tag{9}$$

Eq.(8) contains the oscillator potential, which restricts a motion of a color quark. Besides that we broke the inhomogeneous O(2,6) symmetry with the help of the commutation relations (1). In the special case $\mu\lambda = 1$, $\kappa = 0$ and the commutation relations (2) for SSYA we will obtain more simple expressions for the C^0 and C^{ij}, but the form of the eq.(8) will remain unchanged. Note that eq.(8) can also be applied for a description of a confinement of boson particles such as diquarks and gluons with the same confinement parameter σ.

Let us consider some consequences of this approach for specific color quark characteristics. From the relations (2) it immediately follows nonzero uncertainties for results of simultaneous measurements of quark momentum components. For instance, let $\psi_{1/2}$ is a

quark state with a definite value of its spin component along the third axis. Consequently, $[p_1, p_2] = i\mu_c^2/2$, thus

$$\Delta p_1 \Delta p_2 \geq \mu_c^2/4 \tag{10}$$

and if $\Delta p_1 \sim \Delta p_2$, one gets $\Delta p_1 > \mu_c/2$, $\Delta p_2 > \mu_c/2$. We see that the generalized quark momentum components $p_{\perp 1,2}$ cannot be measured better than tentatively one-half a value of μ_c [7].

One can get an estimation of the σ value using the quark equation (8). As it is seen, M^2 and p^2 entered into the eq.(8) can be considered as current and constituent quark masses squared, respectively. So eq.(8) indicates that the convential relation $p_{cur}^2 = M^2$ for a current quark should be transform to $p^2 = M^2 + \Delta^2$ for a constituent quark, where $m^2 = M^2 + \Delta^2$ is a constituent mass squared. To estimate the σ value with the help of a constituent quark mass m and a current quark mass M values a ground state ψ_0 in a meson has been considered neglecting the orbital angular momentum contribution $L\psi_0$. By this means in ref.[11], where another equation for a generalized quark has been used, it has been obtained that the μ_c (or σ) value is of the order of 0.2 GeV.

Let us estimate the σ value with the help of the value of the confinement rising potential coefficient [12]. One can see from eq.(8) that the coefficient of the oscillator confinement potential for a color particle is equal to σ^2. In the one particle potential approach this coefficient is connected with the so-called "string tension" σ_{str} typically as $\sigma^2 = \sigma_{str}^2/4$, where σ_{str} varies from 0.19 GeV^2 to 0.21 GeV^2 [12]. Hence $\sqrt{\sigma} \approx 0.3GeV$ and $\lambda_{conf} \approx 0.6Fm$. Clearly it is assumed that the conceptions of the asymptotical oscillator potential and the constituent quark are applicable in a confinement domain.

3. Conclusion

Above we considered the model of a color particle confinement with the generalized O(2,6) symmetry in the quantum phase space of a color particle. In the framework of the model the new equations of motion (6) and (8) for color particles have been obtained with the oscillator rising potential which provides the confinement of the particles. The further investigation of properties of solutions of the equations (6) and (8) is the important objective of this model and under consideration now.

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